Vibration Energy

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The purpose of this article is to introduce an energy parameter for judging damage potential on machines from vibration. The historical background of vibration technology has its roots in music, mathematics, military stealthiness, radio electronics, and structural failures. The present application to machines is to correlate vibration to longevity, and to detect wear leading to failure. The parameter of choice has been amplitude, with some considerations of frequency. The frequency has always been in the background as a factor that the analyst applies with some judgment. This article will present a rigorous mathematical theory of vibration energy that fully includes the frequency, in fact, all frequencies. Some benchmarks will be proposed at the end.

Theory

Machines do work by converting energy from one form to another. Some energy leaks from a machine as an oscillation. This oscillation leaking from a machine is the vibration that we measure to judge condition. Much of this oscillation is normal operation. Excessive motion is not good. As analysts, we qualify that motion amplitude by:

a) The mass that is in motion
b) The stiffness of the supports
c) Frequency

Figure 1

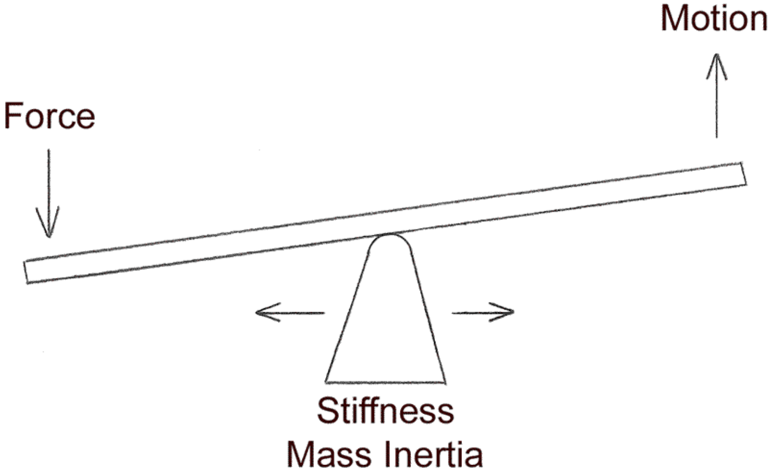


Figure 1 shows a pivoting beam that attempts to portray the relation between force and motion. Frequency is left out. Vibration is a mixture of force and motion of varying degrees. An oscillating force is the source and motion is the result that we measure. Force is not measurable, but an abstract concept. Motion is what we measure. Newton's second law, $F = ma$, provided an operational definition of force, as did Hooke's law, $F = kx$. The measurable motion quantities in these two equations are acceleration and displacement, in a static situation. Strictly speaking, these two linear equations only apply at zero frequency.

In Figure 1, the stiffness of the support structure can move the pivot point to the right resulting in less motion. The inertia of the mass that is in motion can also move it—more mass, less motion. Stiffness is generally considered constant at all frequencies, but is not. Dynamic stiffness is calculated as the ratio of input force divided by displacement, and is plotted versus frequency. Mass is considered constant at speeds much less than that of light, but mass inertia is certainly frequency dependent. Masses do not “like” to shake at higher oscillation frequencies. Mass impedance increases with frequency. Velocity has been a compromise to blend the high frequency of acceleration with the low frequency weighting of displacement. So amplitude alone, as a judgment criteria for vibration severity, is clearly deficient. The overall, or RMS value, has been an attempt to incorporate some frequency into the judgment. What does this have to do with vibration energy?

Energy is defined as force multiplied by motion. Energy = Force x Motion

If the motion is measured in displacement, then energy is work. If the motion is measured in velocity, then energy is defined as power. If the motion is measured in acceleration, well, force x acceleration is undefined. The motion is what we measure. Force is doing the damage, even though we cannot measure it. Energy is the real parameter that provides the “big picture.” We can only measure part of it from physical measurements by these ideal static equations. From a dynamic perspective, vibratory energy is more readily calculated by the conservation of energy principle.

Derivation

In a vibrating system, there is a continuous exchange of potential energy with kinetic energy, but the total remains constant. The maximum potential energy at the extreme displacement of the spring equals the maximum kinetic energy of the mass flying through the neutral position.

$P.E._{MAX} = K.E._{MAX}$

$P.E. = \frac{kx^2}{2}$ k = stiffness
x = displacement

$K.E. = \frac{mv^2}{2}$ m = mass
v = velocity

Total Energy U = P.E. + K.E.

Both potential and kinetic energies are a function of position. At maximum motion, kinetic energy is a maximum and potential energy is zero. At the extreme displacement position (90 degrees later or earlier in the cycle) potential energy is maximum while kinetic energy is zero. Take one extreme condition when potential energy is maximum and kinetic energy is zero:

Total vibratory energy $U = \frac{kx^2}{2} + 0$

Substituting Hooke's law for k, ($k = \frac{F}{x}$) and Newton's second law for F, (F = ma) and the conversion from displacement to acceleration: x = displacement = $c\frac{a}{f^2}$ Where c is a constant for conversion.

The total energy of vibration is:

$U_{total} = c\frac{ma^2}{2f^2}$ = Vibration Energy (VE)

c = 1 if using SI units
m = Mass, Kg (Total mass of vibrating object or machine)

a = Acceleration, m/s^2 peak
f = Frequency, Hz, s^{-1}

The resulting units are $Kg \cdot \frac{m^2}{s^2}$(mass x velocity squared) which is Nt · m or Joules.

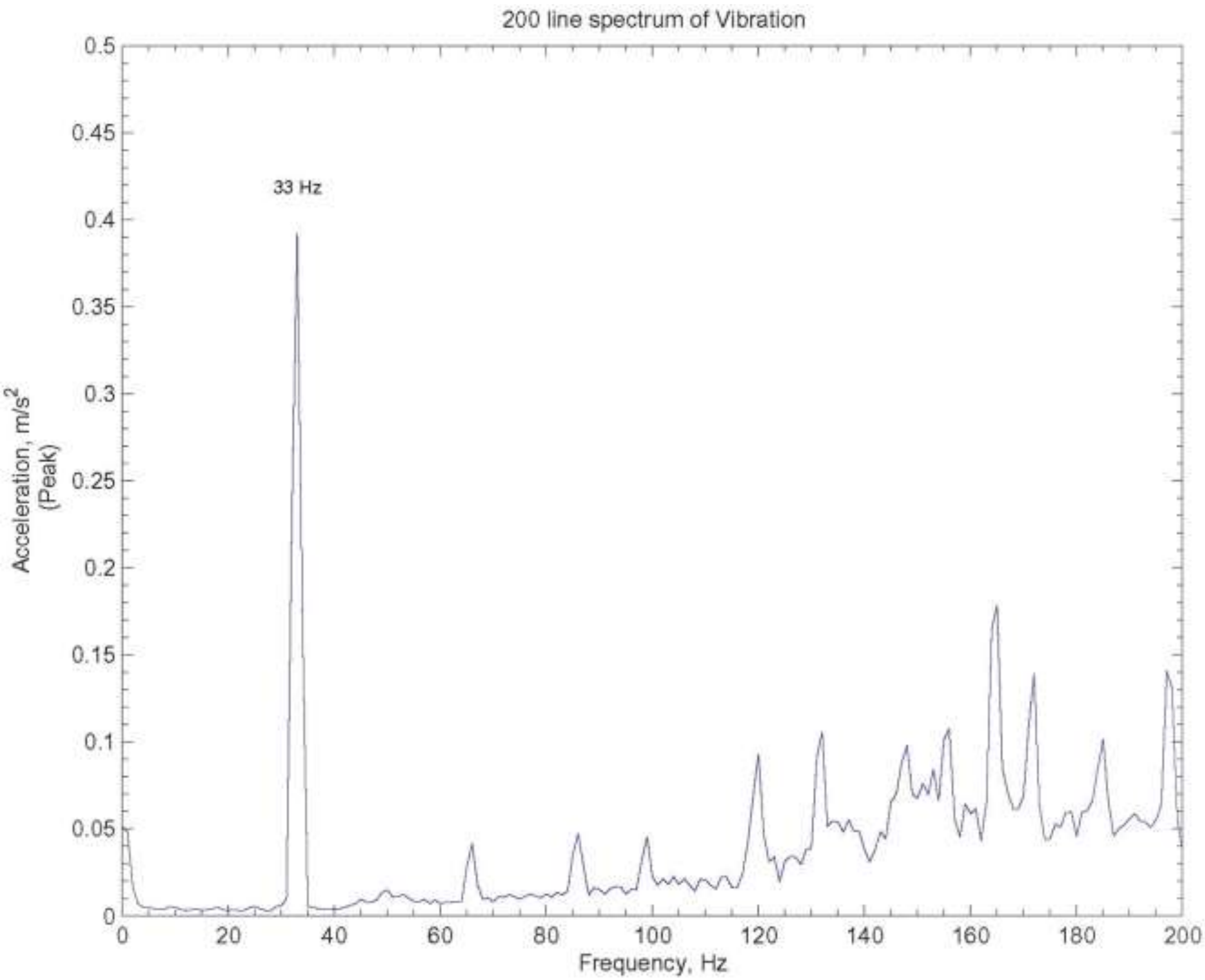
The energy of vibration is proportional to the mass in motion, proportional to the square of the acceleration, and inversely proportional to the frequency squared. Acceleration and frequency can be measured directly with FFT spectrum vibration instruments. The mass in motion will need to be estimated. It is the total rigid body motion.

To obtain a complete picture of the damage potential, this calculation must be done at each frequency, and summed. This is easy for a digital signal processor that has the amplitude stored at each bin frequency. Therefore, the total energy should be summed from zero frequency to some maximum frequency of interest, F_{MAX}.

$$VibrationEnergy(VE) = \frac{cm}{2} \int_0^{F_{max}} \frac{a^2}{f^2}\, df$$

Figure 2 is a vibration plot of acceleration, m/s^2, versus frequency in Hz. The full span frequency is 200 Hz and 200 lines are displayed, so each bin is 1.0 Hz wide.

Figure 2



For a machine on springs, the mass is everything above the springs. For a machine bolted down, it is the total connected mass to the softest joint, which could be everything above dirt. For a proximity probe measurement, the mass is the rotor, but the proximity probe displacement will need to be converted to acceleration.

If the vibration goes non-linear, or there are metal impacts, then we get harmonics and high-frequency shock pulses with broadband energy. These get accumulated into the total energy value.

There are several assumptions to this theory:

1. Vibration energy is related to wear, or damage.

2. The relationship is linear. That is, the ratio of energy leakage (that we measure as vibration) to damage is a constant.

3. This linear relationship remains constant over time. (Temporal symmetry)

4. Measurements are taken at, or as close as possible, to the bearings.

Applications

The measured quantities, a and f, are already resident in the digital table of an FFT spectrum analyzer as bin values if the vibration was measured in acceleration. This is an arduous calculation for humans by hand, but easy for a computer to do quickly.

This quantity, VE, should be programmed into future vibration instruments for a direct measure of vibration energy and damage potential. The estimated mass could actually be combined with the constant as a relative number for trending of periodic measurements in a routine machine health monitoring program.

This total vibration energy, VE, is similar to an overall, but is an energy product considering mass and frequency, not just an AC signal RMS value. This value was calculated for several hypothetical examples, and is shown in Table 1. The energy value is given in joules, only because that results naturally when the mass is in Kg, the acceleration in m/s^2, and the frequency in Hz. In English units, the energy value will be ft-lb_f with the g_c value conversion.

$$g_c = 32.2 \frac{ft-lb_m}{lb_f-sec^2}$$

Table 1. Vibration Energy Calculations for Several Hypothetical Conditions

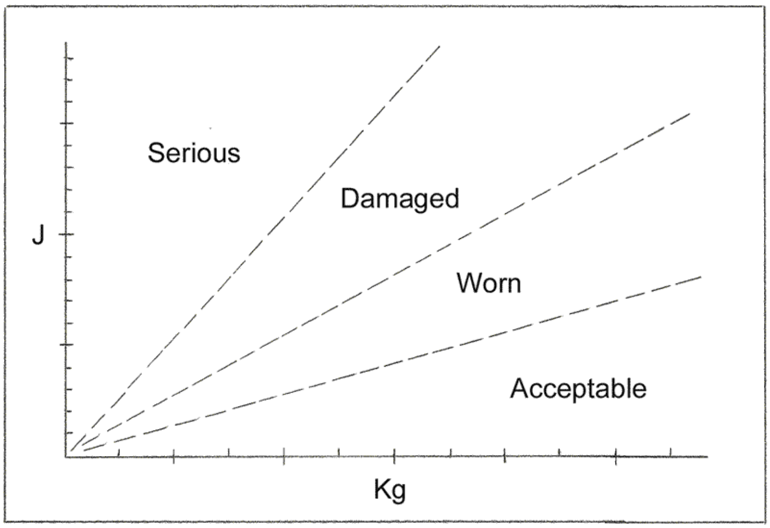
Condition				**Vibration Energy, Joules**
Mass, Kg	**Acceleration**		**Frequency, Hz**	**U =** $c\frac{ma^2}{2f^2}$
	m/s^2	***g***		
200	0.41	.042	20	.042
200	0.83	.085	40	.042
200	1.24	.126	60	.042
1,000	0.41	.042	20	.210
10,000	0.41	.042	20	2.10

The conditions in Table 1 are all fans mounted on springs that measure .09 in/sec RMS velocity, which is the American Society of Heating Refrigeration and Air Conditioning Engineers (ASHRAE) balance upper limit. The first three are a 200-Kg fan at three different speeds. The energy calculation remains the same. The last two are fans that are more massive. These were energy calculations at a single frequency, the filtered amplitude at 1 x rpm. If the machine had some mechanical distortion, such as from misalignment, then harmonics will be present with amplitudes at higher frequencies. A worn bearing will be producing high frequency broadband energy from shock pulses. These amplitudes will be additive and tally to a larger energy number.

Limits

Based on these preliminary (and hypothetical) calculations in Table 1, it is possible to introduce some limits for initial judgment. The ASHRAE balance specification of .09 ips RMS, suggests an energy value of 0.05 J if only a pure unbalance is present on a moderate size fan on springs. For larger machines, this number increases to approximately 2.0 J or more, which could be a general industrial fan on a concrete pedestal. This suggests that an appropriate specification for vibration energy should be normalized to the mass, as depicted in Figure 3. No limits are shown in Figure 3, as that would require a more extensive statistical study correlated to descriptions of serious, damaged, worn, and acceptable.

Figure 3



This theory of vibration energy, normalized to mass, could be used as a starting point for a long sought after field balance specification. Present balance specifications focus only on the filtered amplitude at rotating speed, while ignoring all other frequencies (such as harmonics) and the mass in motion. Unbalance, we know, causes damage, which is related to amplitude, but not entirely. Amplitude is one factor. Other factors are the mass in motion, the size of bearings and how much they are complaining. Vibration energy is a better metric to quantify economic loss potential caused by unbalance.

This calculation of energy from a frequency spectrum is already done in acoustics in the form of sound power.

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