Symmetry, as it relates to dynamics, structures, and machines

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Introduction

Symmetry is a poor design doctrine for structures. The reason is resonance. Symmetry, however, is desirable for machine functionality. This article will explore the origins of our flawed principle of arranging structures in geometric patterns for aesthetic appeal. It will also explain how the geometry of parts affects machine operation and some philosophical considerations.

Symmetry is defined[1] as --

"beauty of form arising from balanced proportions" or

"the property of remaining invariant under certain changes"

The operative principle of symmetry is replication. An exact copy is repeated again and again in a linear or spacial motif. It can also be a repetitive pattern in time where the same event re-occurs.

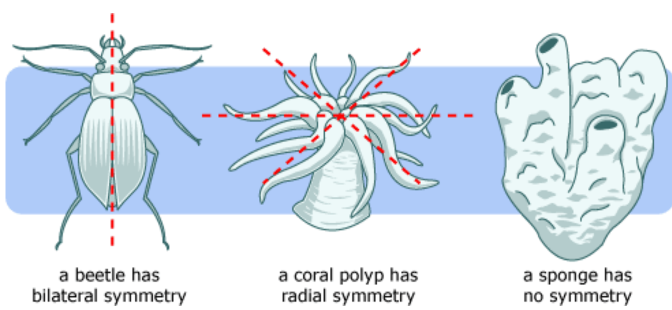

Figure 1: Symmetry in biology

Bilateral symmetry in animals, as in Figure 1, is a common example of "reflection symmetry," where a line going through the object divides it into two pieces which are mirror images of each other. Radial and spherical symmetry are also common in biology. There is also spatial symmetry, particularly in crystal structures, where geometric shapes are repeated in 3-dimensions, and temporal symmetry where relations hold true for all time. Spacial symmetry implies also that a circle remains a circle when transported to another galaxy. Temporal symmetry implies that it will remain a perfect circle forever. These transformations in space and time are the basis for symmetry in mathematics. The conservation laws of physics rely heavily on spacial and temporal symmetry, and mathematics is the language of science

The origins of the concept of symmetry may be observations of the cosmos by prehistoric peoples. The ancient Greeks, specifically the Pythagorean's in the 5th century B.C., coded symmetry into their mathematics, music, and philosophy. The

1 Merriam Webster's Collegiate Dictionary, Tenth Edition, 1995

five Platonic solids shown in Figure 2 are regular polyhedra with straight sides, equal lengths, and equal angles.

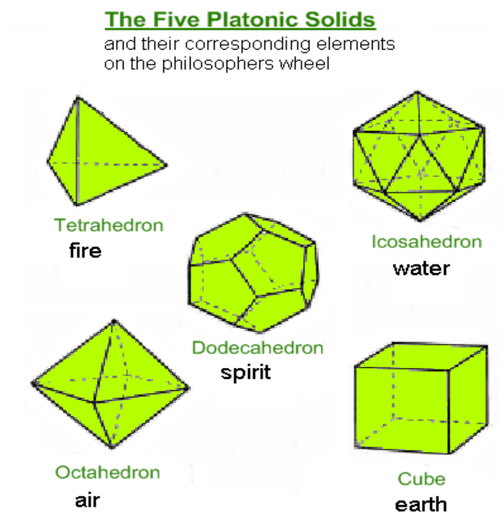

Figure 2: Platonic solids

The Pythagoreans were so obsessed with this discovery, that to them the structure and operation of the cosmos was in harmony with whole numbers. The search for order is the basis of human psychology for comfort in understanding nature. That brings up the question of "Does nature have a purpose?" If it does, then that implies that symmetry is a valid foundation for the structure and operation of nature. If nature does not have a purpose, then that implies that it is chaotic.

Mathematics & Philosophy

Humans are pleased with patterns. I am not sure whether my dogs even care. Straight lines and round circles are shapes that we strive to attain when designing structures and machines, Figure 3. Symmetry in static design of structures is convenient when placing lines on paper or computer aided drawings. It is easier to duplicate when constructing the building or fabricating machine parts. It lowers the cost of construction.

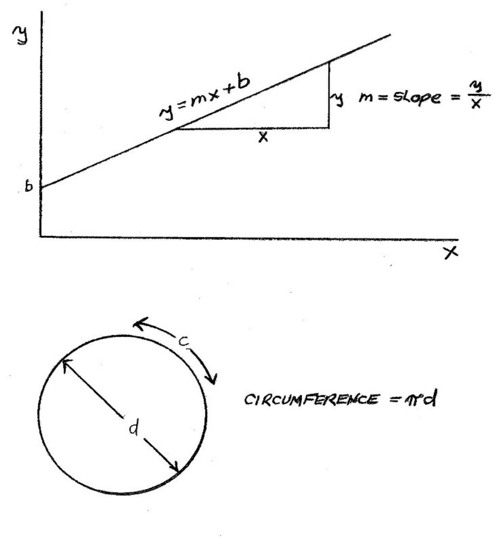

Figure 3: Perfect geometry

We have even developed a vision of these perfect shapes and mathematical formulas to describe them flawlessly. If we could only repeat these patterns endlessly to replace the natural world, then we would have created the perfect utopia.

We also like dualities: left - right
black - white
on - off
right - wrong
good - evil

There are no such polarization's in nature. We have created them with our thinking. The natural world is not digital, at least not on a macroscopic scale.

Symmetry implies orderliness in space and time. This supports the philosophy of determinism as framed in the "clockwork universe"where the creator wound up the clock, thus providing it initial energy, and it will run in that repetitive pattern forever. The alternative philosophy is that humans have a free will to chose right from wrong, and have the capacity to alter the future, up to a point. In between is the philosophy of "causal determinism", where every event has a cause. There are events around us that we have no control over, but do affect our choices. Epictetus, Figure 4, a Greek Stoic philosopher in the 1st century B.C., taught that all external events are beyond our control and we should accept whatever happens. The only thing that we really have control over is our attitude. So don't expect the future to repeat, or for the circle to remain round. Various religions are framed on these concepts.

Figure 4: Epictetus, born a slave, tortured by his Roman master and lamed, famed Stoic philosopher.

The causal laws of determinism form the basis of science, and adheres to the idea of symmetry. As individuals, however, Epictetus taught that we are responsible for our own actions which can be controlled through self-discipline. Free will is consistent with the world's major religions, where a person can make good and bad choices. Free will is a departure from symmetry, and strays from the laws of science. Asymmetry, which is the opposite of symmetry, allows for choices to be made, and the future remains unknown. This appeals to humans who cling to the ideas of freedom and free choice. Symmetry also appeals to humans who desire order. So here we have a dichotomy where symmetry is pleasing to the visual senses, easy to predict, and cheap to produce, but humans do not want to be controlled. Free will is counter to determinism and implies disorder. This tug-of-war between order and chaos is one area where science and religion still debate. They both adhere to order and symmetry, and both allow for some disorder (choices). Nature is not under anyone's' control. These concepts were made clear by Epictetus, who promoted "live according to nature." That is, make choices consistent with lack of control of the natural world. The dynamic physical world does not care. We as humans have an aesthetic appeal to the symmetry of patterns and perfection. The physical external world around us does not care about perfection, even though the biological and chemical world does produce patterns at the microscopic level.

Cultural development over time has created patterns of behavior that strive to integrate the chaotic with orderliness. Origami is the ancient Japanese culture of paper folding. It mimics some natural plant unfolding patterns. It highlights symmetry in the natural world. Music is clearly an expression of symmetrical tones. Farmers tended to plant straight rows.

In mathematics, symmetry has a more precise definition–that an object is invariant to a transformation. That said, the Pythagorean formula remains as $c^2 = a^2 + b^2$ when moved to the surface of Jupiter and remains so until the 35th century and beyond. This clearly is not favorable to the young woman trying on a new dress, or doing her hair. She is seeking a transformation of some kind. So from this perspective, we can conclude that humans are asymmetrical in behavior patterns, by choice. In a dynamic world, symmetry can do damage when combined with wave motion, as will be demonstrated later.

Biology & Chemistry

Biology has favored symmetry from the beginning. The double helix pattern of DNA, Figure 5, is clearly a repetition in space of smaller units.

Figure 5: Double helix model of DNA

The structure is repeated, but the specific pattern of the details is variable. Every blade of grass is different from another, even though they all share the same DNA structure. This served the purpose well to duplicate a complex organism in both structure and somewhat in behavior, while allowing evolution to proceed with variants. The microscopic structure of DNA is obvious, but how does this translate into patterns of biological organisms in the macroscopic world? The answer may be in "movement."

For animals to move in three dimensional space, they needed to go right or left, and up or down. This necessitated organs on both sides to push against the environment. The result was bilateral symmetry, Figure 6, a definite head and tail, and a top side and bottom side. Anything else would be perceived as a distortion. People prefer symmetry in physical attractiveness also and have extended this appeal to music and the arts.

Figure 6: White parrot bird

It takes energy to create symmetry in biology. So this represents a higher level on the energy ladder. Chemistry also favors symmetry on a microscopic scale, but at a lower energy state. Crystals arrange themselves into patterns that represent minimal energy states, Figure 7.

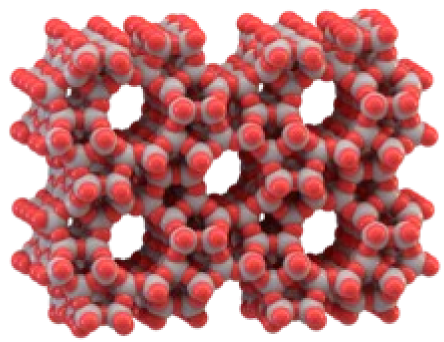

Figure 7: Zeolite crystal

Just as water flows downhill, materials naturally move to lower energy levels when undisturbed. That condition can be changed with the addition of organized energy. Crystal growth produces symmetry only in grains, but does not continue for larger material structures unless organized energy is added.

There is another form of energy, namely heat, that is disorganized. Thermal energy is chaotic. It destroys symmetry by agitating molecules to random orientations. It disperses material in the form of fire, wind, and wave motion. These are all various forms of heat energy. This brings up the 2nd law of thermodynamics. Where does it stand in relation to symmetry and the order of things? Generally, the second law points toward asymmetry of spacial order and in time, primarily because of friction and "lost" energy that is translated into heat. If left alone, all things will naturally re-arrange into random orientations at a minimum energy condition. So then, the great philosophical question for the cosmos, still unanswered, is "Why the universe has not turned cold and dark?" as predicted by the 2nd law. Suns continue to be formed, adding heat energy to the mix.

Physics

With this background, we can now discuss how symmetry affects the dynamic behavior of structures and machines. Symmetry in physics has been generalized to mean invariance, that is, lack of change. Symmetry has been a fundamental concept in classical physics and is the backbone of the four conservation laws --

1.Conservation of linear momentum of a system

2. Conservation of angular momentum, or spin

3. Conservation of energy - the 1st law of thermodynamics

4. Conservation of mass

For these laws to be valid, friction is assumed zero, a closed system is assumed with no external forces, and invariance is assumed. Invariance, in space and time, is closely related to symmetry. Invariance is an idealization - a fanciful concept that appears attractive to the human mind in a chaotic natural world. If things vary in an unpredictable way, then symmetry is not possible.

Imagine, for a moment, that all of nature transformed into perfect symmetry. Imagine a place where the temperature never changed, where every fish weighed exactly the same, and where every rock is a perfect sphere. That does not coincide with our observations. It would be a very uncomfortable environment for humans. Yet we attempt to build uniformity into our structures and machines with symmetrical shapes. We desire it as some form of beauty and comfort. The natural world abhors these attempts. Nature guarantees that the next shape will be different, and the one that we produced will change over time. Nature further rebels by generating oscillatory motion, which is a natural motion, but when combined with a symmetrical shape, generates either beautiful tones or horrible damage.

Symmetry is characteristic in biology and the microscopic world, but it is not characteristic in the macroscopic natural world, Figure 8.

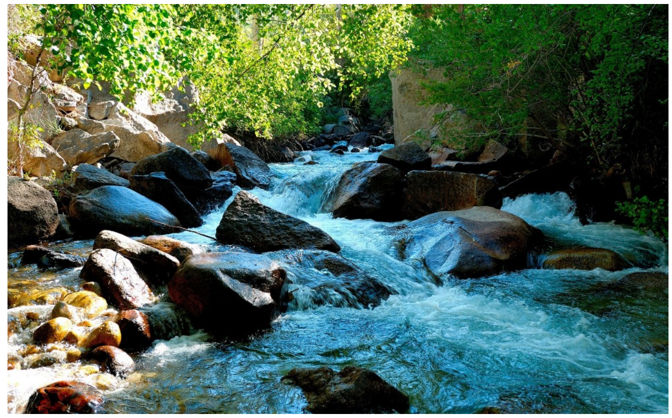

Figure 8: Nature is chaotic

Natural materials are wide band in frequency response. Rocks, water, and air structures do not respond to frequency with any appreciable amplification. They are not frequency selective. We arrange them, with our design of patterns, to become frequency selective, intentionally or not.

The sources of energy in nature are also not frequency selective, i.e., heat, wind, gravity, lightning, and chemical are all broadband energies. These natural energy sources can interact with the natural materials without creating serious oscillations, with the exception of earthquakes. Gravity, however, does produce spherical symmetry on large material bodies up to a certain size. Larger structures in the cosmos, like galaxies, lack this symmetry.

When we look at nature, we tend to seek out patterns. Basic symmetries in nature underlie the various conservation laws. We would like the laws of physics to be constant at all places for all time. P.W. Anderson in his 1972 article “More is Different” stated "........physics is the study of symmetry." There is more chaos at the subatomic level, even though physicists continue to seek patterns of orderliness. Now, in the 21st century, obscene amounts of resources are being expended in pursuit of "super symmetry", where every matter particle has a corresponding force particle. What would happen to the laws of physics if spatial and temporal symmetry proved to be invalid assumptions? If those assumption were found to be weak, then the laws would become local to regions of space and time.

So these are some of the underpinnings of our psychology for arranging material to serve our purpose. We like order and dislike disorder. The consequences of arranging materials into symmetrical patterns is that we create structures with modes of behavior that amplify oscillatory motion at specific frequencies. In a dynamic world, symmetry is a poor design principle because it supports resonances of beams and plates, it generates pulsations from blades and vanes that pass stationary edges, and it promotes weaker and softer flexible structures.

Resonance

Uniform thickness of homogeneous materials (which do not exist in nature) and geometric patterns support resonance. Flat sheets drum, Figure 9.

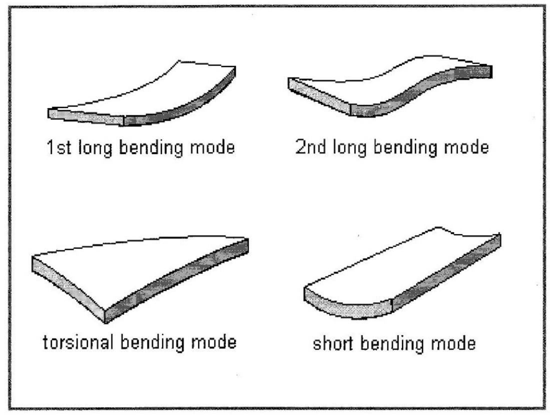

Figure 9: Drumming modes of plates

Regular cross sections behave like vibrating strings with beam modes. They have "natural" frequencies. The issue is that they become frequency selective when we arrange them into regular geometric shapes. Resonance problems in structures and machines are basically symmetry related and are human caused. One solution is obvious – delete symmetry. If we chose to continue to design symmetrical shapes, then we are left with modeling and testing to move the modes to frequencies that will not be troublesome. If we chose to not design out resonance, then we are left with the five known field fixes for resonance. These are:

1. Change speed
2. Change the natural frequency of the responding part
3. Add damping
4. Reduce the source input energy
5. Dynamic absorber, also called auxiliary mass damper

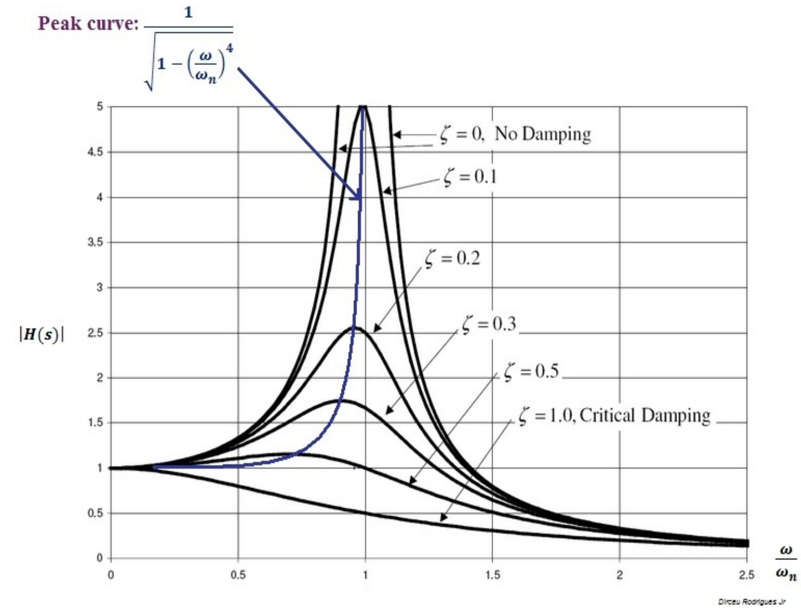

Figure 10: Resonance plot with damping.

Figure 10 is a resonance plot with damping of a single degree of freedom system that is frequently used to model "real" systems in order to visualize and explain corrective methods. The vertical axis is motion. It can be labeled amplitude ratio H(s), transmissibility, or just plain displacement, velocity, or acceleration. The horizontal axis is frequency normalized to the natural frequency ω_n. Important observations are:

1. Newton's second law, F = ma, only applies at zero frequency where the motion in acceleration is truly the ratio of force divided by mass, a = F/m. At any other rate of oscillation, the acceleration becomes a function of frequency.
2. With critical damping, the motion is almost linear, decreasing with increasing frequency. Critical damping is where there is no oscillation. The object is displaced and slowly oozes back towards the starting position.

The non-linearity of this resonance behavior with any damping less than 1.0 creates problems with mass balancing, fatigue cracking, motion sickness, and other undesirable effects on structures and machines. There is, however, a blessing to be had in this phenomenon of nature when we want amplified motion. We can create pleasing sounds with music and accurate time pieces.

Structures

There are many ways to design out resonant behavior in structures. One was alluded to above with non-uniform thickness, Figure 11.

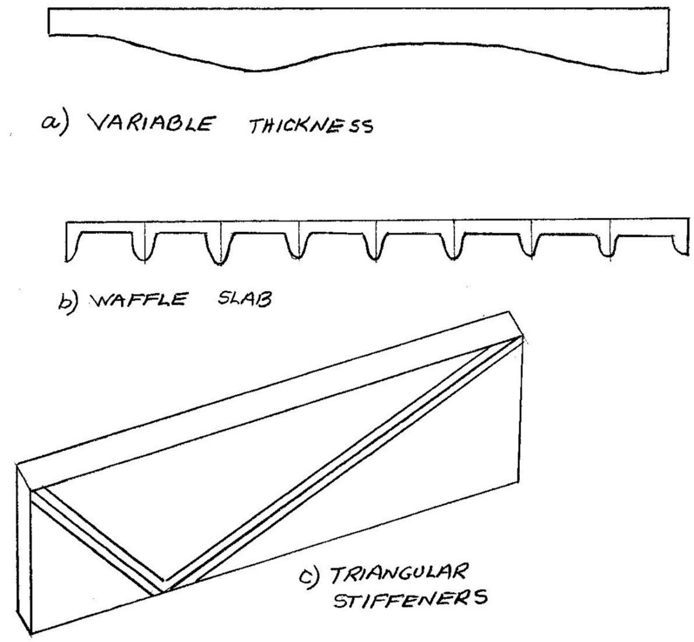

Figure 11: Breaking up the symmetry.

If the thickness of a plate is varied, then it will respond with a different mode or not respond at all to specific frequencies. If a beam is varied in cross section along its length, then it will not respond in the same manner as a uniform beam. Flat plates and uniform cross section shapes are easier to produce at the mill factory and less costly. They are also easier to fabricate with. Non-uniform plates and variable cross section beams are generally not available for construction but are produced in the form of castings, forgings, moldings as in plastic injections, and composite layups as in fiberglass skis and boats. Buildings with non-uniform thickness panels would be a custom design that would be fabricated on-site, but not unheard of. Southwestern pueblo adobe construction falls into this category, Figure 12.

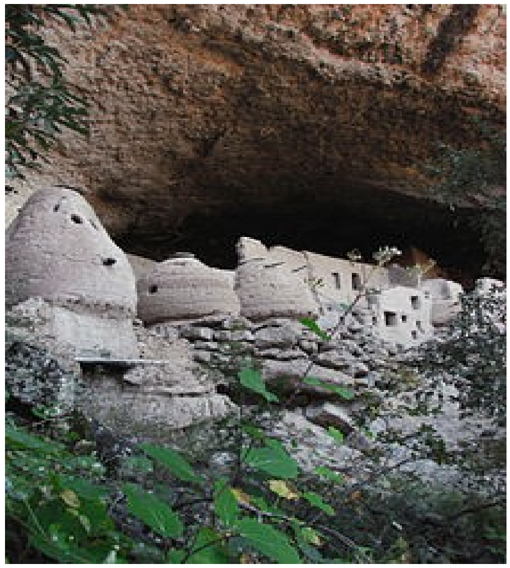

Figure 12: Southwest adobe cliff dwellings.

A second method is to use non homogeneous materials, Figure 13.

Figure 13: Composite.

Materials that are not of a constant density throughout are not frequency creative. Composites of resin and cloth are a good example. They have good damping properties because stress waves must cross boundaries of different density materials and are attenuated along the path with reflection and absorption at each boundary. Concrete is another example. Concrete is a composite of sand, lime, and rocks. Composites can respond with frequency specific modes, but the amplification is much less.

Laminates are another material structure that is non homogeneous. Layers of different density sheets will bend when exposed to forces, creating shear strains at the interfaces, and dissipate the energy of motion into heat.

A third method is to vary the shapes. Curved panels are stiffer and do not drum. High speed vehicles of metal outer skin construction have evolved into curved shapes for aerodynamics and for quietness. Bent sheet metal is stiffer and less resonant. Corrugated roof decks are somewhat non flat, and stiffer.

The problem shapes are those with long linear dimensions. A stiff string will have a tone, depending on it's length, like a cello string. A structural beam will also have a tonal frequency, but it may be too low to be audible. A two dimensional plate will drum. Three dimensional shapes would be superior. A body with no long linear extension, such as a cube or a round ball, cannot drum. How to achieve this in a building envelope is a challenge. Owners and architects want to maximize tenant usable space with large cavities that can be occupied. A round ball is difficult to make "sing."

Irregular perimeter walls and ceilings create phase mismatch when reflections from those surfaces mix. Acoustic engineers know this principle well in designing interior spaces, Figure 14.

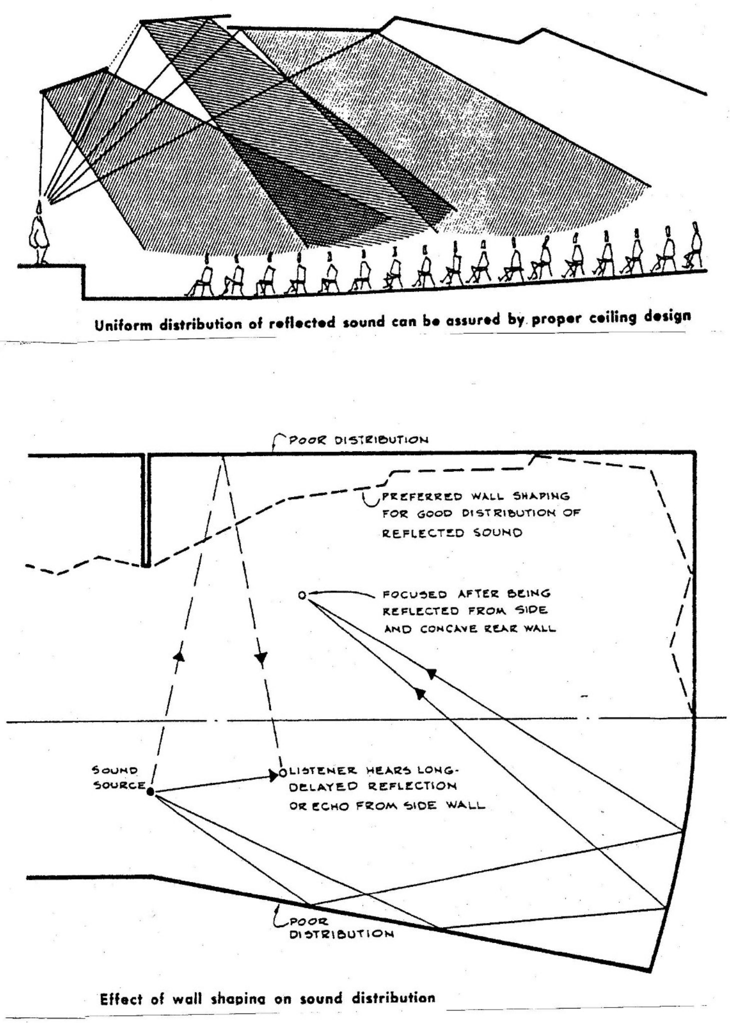

Figure 14: Interior space design.

The idea is to break up surface flatness to promote diffusion. Flat surfaces can support standing waves within an occupied space when the reflections constructively interfere.

A fourth method is to place the support points at irregular spacing, Figure 15(b).

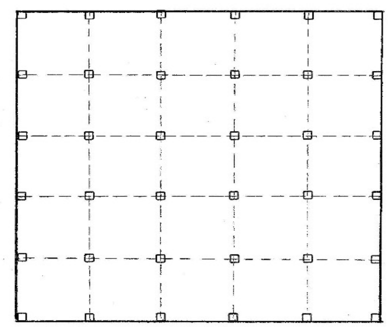

FIG. 15 A. REGULAR COLUMN SPACING AND FLAT EXTERIOR WALLS

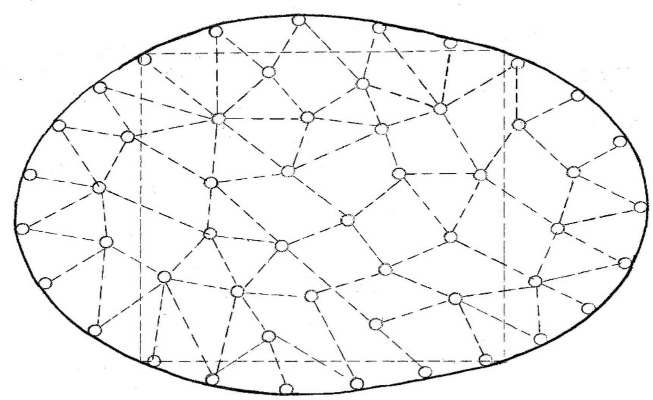

FIG 15b. RANDOM COLUMN SPACING AND CURVED EXTERIOR WALLS.

Figure 15: Column Spacing (a) Regular and flat exterior walls, (b) Random and curved exterior walls

Structural engineers tend to space the column supports at regular distances because that is easy to replicate on a drawing. It also requires less thinking on the construction site when ordering materials and managing work crews. A variable column spacing would give the building protection from cross talk of panel drumming modes. This principle can be extended to three dimensions by placing column attachments at different levels. This sounds unconventional, but has proven to work very well on machinery when supporting an engine or transmission, Figure 16 for example.

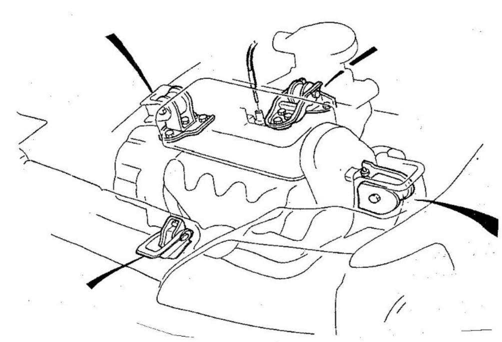

Figure 16: Three dimensional support for machines.

In summary, chaotic structures have broadband frequency responses. They do not amplify the input energy with oscillatory motion at a specific frequency. They are "untuned."

Machines

In the previous section, symmetry was portrayed as detrimental to structures. In machines though, symmetry is necessary for proper functioning. Machinists go to great efforts to produce precision geometric shapes into the components of machines. It started with the wheel. Machine tools were based on round bearings, Figure 17,

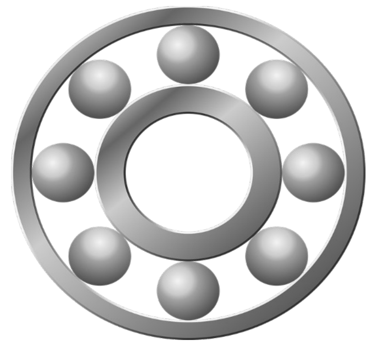

Figure 17:Ball bearing.

and precision ground flat ways, Figure 18.

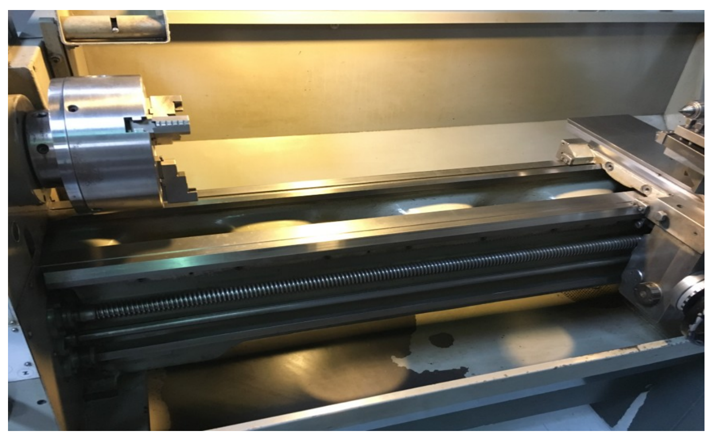

Figure 18: Machine tool slides on a lathe; precision ground to be flat and straight.

To produce flat sheets and round shapes, the machine tools must repeat. That means that there must be good bearings, flat guide surfaces and precision rolling element bearings that can repeat the rotating center. The wheel is a good example that rides on a flat road surface for comfort. Bearings need to be round to support circular motion, but there is no reason for the remainder of the shaft to be round, other than it is convenient to shape that on a lathe.

Rotating machines (motors, pumps, fans) generate strong centrifugal forces. Things operate much smoother when the rotating center, as defined by the bearings, is concentric with the mass center, as defined by the geometry of material distributed about the rotating center. That means good balance. Figure 19 shows a tire with a balance weight to "adjust" the mass center.

Figure 19: Tire with balance weight.

Mass balance is a form of radial, or circular, symmetry. This is true for rotating machines that we build. In the natural world, centrifugal force, or more accurately, centripetal force, does not force symmetry on astronomical bodies. Rather it generates elliptical paths of motion.

The preceding discussion about rotating symmetry is valid for the rotating parts of machines, but does not apply for the structural supports. The structure that supports the machine does not rotate. The structure does not need to be symmetrical. The base does not need to be a rectangular shape. It can be triangular, or elliptical, or any other irregular shape, Figure 20.

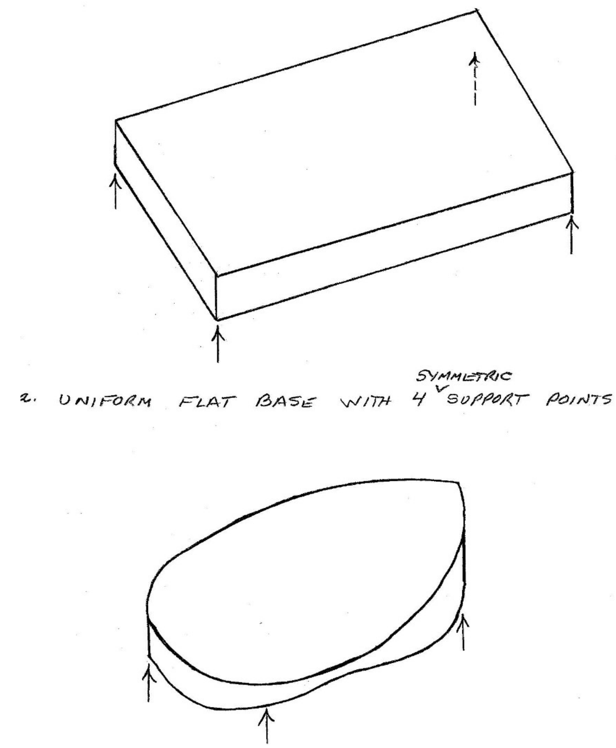

Figure 20: Bases for machines (a) symmetrical and (b) asymmetrical.

It can be of non uniform thickness. The support points can be staggered. The enclosure can be non uniform thickness panels, or composites. The fact that we have rectangular bases with four support points at the corners must be a hold over from the shape of wagons. In fact, it is well known that three support points is more stable than four. The evidence is optical machines, like theodolites and precision measuring tools, that are supported on a tripod, Figure 21.

Figure 21: Tripod 3-point support for stability.

There is no good reason for machines to be on four support points, and very good reason to be placed on three, as long as the center of gravity is within the footprint. There is even good justification to arrange the support points to be on different elevations in three dimensions. That way, the center of gravity can be below the support points and provide an added level of stability to external forces. The obvious example is watercraft that have the center of gravity below the center of buoyancy.

Figure 22: Satellite

Aerospace structures, Figure 22, in the absence of gravity, are not confined to downward forces only, and have supports points placed strategically where space allows. Satellites, and other aerospace structures, may have started being symmetrical, but have evolved into non-symmetrical configurations for functionality, with the added benefit of resistance to vibration during rocket launch.

A summary of better designs to avoid resonances are --

1. odd number of blades
2. vary blade spacing
3. vary diameter of shafts
4. contour shapes with smooth curves (car bodies)
5. circular or elliptical shapes rather than rectangular
6. honeycomb
7. laminates or composites
8. many joints to attenuate transmission paths
9. convolutions or bent ridges
10. changes in impedance of material stiffness (hard to soft)

Conclusion

Flat panels, uniform cross section shapes, and geometric spacing is easy to build with in the factory and easy to assemble in construction or fabrication of machines. We have created our own resonance problems by insisting on symmetry in design of structures and machines. Asymmetry of materials and structures is more natural and less likely to support drumming and string modes. Spherical and circular symmetry is beneficial in machines for balancing, and absolutely necessary for good bearings to operate with precision. Linear and planar symmetry is detrimental for structures.

I propose that a mental state of chaos, with variability, will open a window into a better understanding of natural phenomena.

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