Mathematics via Tao

The question to be explored in this section is "What is the connection between mathematics and reality?" This question was presented to a group of friends during a dinner party after being slightly imbibed. These were professionals with backgrounds in mathematics. One mature male piped up quickly with the answer "capitalism". A very astute reply that will be elaborated on later.

The science of mathematics has it's roots in building, music, astronomy, oscillation, and philosophy. It was the ancients who needed to size structures for occupancy, or just for aesthetics, that developed geometry. It was ancient musicians who strived to produce pleasing tones that gave us the beginnings of harmonics and waves. It was ancient astronomers that gathered numbers about the motions of heavenly bodies. And it was the Greek Pythagoreans who saw a mystical beauty in proportions. In this article I have intentionally avoided including any formulas, but rather chose to focus on the philosophy of numbers.

Arithmetic

We were formally introduced to numbers in the third grade class of arithmetic. Our mothers may have counted with us earlier as infants with toys. Counting comes early in the human development. Arithmetic is all about counting to establish a quantity. The quantity could be the number of sheep, the weight of grain, the height of a tower, the number of wives, or the pile of gold coins. The counting of money, or anything else, establishes wealth. Arithmetic is the mathematics of the rich and poor alike, but mostly for those who perceive themselves as more important, and wish to project that image. So arithmetic was a convenient invention for the capitalist as it related to money. It was clearly an invention, and firmly connected to one's standing in the society.

The old debate was, and still is, whether mathematics was an invention or a discovery. In the case of arithmetic, it was probably the discovery that we had ten fingers and ten toes. Most all societies now use the base ten number system using Arabic symbols. The Arabs developed this system after borrowing it from India. A base five would have been just as valid after observing that we have five fingers. This base ten lends credence that arithmetic was a natural discovery that was adopted to be an invention for the benefit of trade. It is probably safe to surmise that counting came before money, and that money would not have been possible without the tool of arithmetic.

A future development in arithmetic came centuries later in the form of statistics. This allowed numbers to be organized in a form for decision making. Organized data then becomes information. That information was especially useful for administrators as a tool for future planning. By measuring a population and calculating the average, or mean, a government official can better understand his or her subjects. This has led to better governance, or worse governance, depending on how it is interpreted and used.

Geometry

Geometry is the math of construction, especially triangles. This was clearly an invention because triangles do not exist in the forest or among the rocks. The Pythagorean formula is not found anywhere in nature. Without geometry, the Egyptians could not have built pyramids, the Greeks could not have built ships, and the worlds great religions would not have their temples of worship.

Circles can be visualized when looking at heavenly bodies. Astronomy provided the symbolism for the circle. A perfect circle does not exist in nature, but the mathematics of a perfect circle was codified by the Greeks with the definitions of the circumference, the radius, and the transcendental number π. Further developments of the conic sections of ellipse, parabola, and hyperbola proved later to be very useful for describing motion. So geometry, being an invention, opened a window into the natural world.

Trigonometry

The mathematics of the triangle introduced the sine, cosine, and tangent. These are static functions useful for building and for calculating the dimensions for a manufactured product. A more powerful extension of these functions came in the form of understanding wave motion. Waves are a natural oscillation ubiquitous in nature. Repetitive motion naturally occurs as a periodic rise and fall in the positive and negative directions. Tree limbs sway, birds flap wings, ocean waves lap, planets rotate, light has a frequency, and sound fluctuates air molecules.

These natural motions can be mathematically quantified by taking a leg of a triangle and rotating it 360 degrees. The rotated leg becomes a vector that repeats with a value of amplitude ratio-ed to the sine of the leg. So the sine leg of a triangle now becomes a number moving around the circumference of a circle tracing out a frequency and an amplitude representing the oscillating motion. How beautiful an analogy. Humans like this kind of symmetry. The trigonometry of sine waves becomes the math of natural vibrations.

Algebra

Algebra is the math of engineering. It attempts to simplify motion by making it static and linear. Engineers like to keep things simple, not only because they like to, but because of necessity. The natural world is not linear, but non-linear math is unwieldy and can give a person a big headache. To be able to move forward with a task, the applied scientist (an engineer), needs to estimate to arrive at a number that is not perfect, nor accurate, but close enough for the job. They make assumptions to remove the small variability in the phenomena to see the big picture. The goal is to finally have a linear equation with only one unknown variable that can be calculated with the algebraic equation. Stuff gets done with algebra.

Algebra does not represent the real world, but an approximation to reality. It functions as a tool to make all the goods that we enjoy in our modern world. To make things even worse, many engineering equations have coefficients, which are an admission to our ignorance. A coefficient in an equation is a "fudge factor" to compensate for the lack of complete knowledge, but it makes the math work. We like linear equations with coefficients. It gives us the illusion that we know something about nature.

Calculus

The fundamental problem in science is describing change. Calculus attempts to do just that. Calculus is the math of change. It is a further abstraction from any of the previous mathematics described above. There is differential calculus and integral calculus.

Differential calculus is a study of the rate that quantities change. Integral calculus is a sum used to calculate the area and volumes of shapes. Let it be known that calculus has functioned to drop out students from college education more so than drugs, sex, or running put of money. It is no easy subject to grasp, but in the hands of a competent mathematician, can do wonderful manipulation of numbers to calculate the trajectory to send a spacecraft to Jupiter.

Binary Simulations

Imagine a large and expensive apparatus for testing with numerous sensors that stream zeros and ones into a digital memory from which is extracted a model of what is going on within the machine. This is attended to by a staff of engineers and scientists that want to see some desired outcome. It is very expensive with high expectations for results. This is supposedly another window into reality. Let's dissect this.

First, this test is designed around a theory, or hypothesis. Some assumptions have to be made. The assumptions may or may not be valid. Some of the assumptions may be about linearity. Others may be estimates for unknowns. Still others may be just pure guesses. The objective is to run the test, extract the numbers, load them into the model, and see if the results match the expectations. If not, we may tweak the model until it converges to a favorable match. We may even animate the test results to visualize what is going on, and perceive this as a window into reality. Now we can debate whether this is real or are we deceiving ourselves. These digital simulations could be a rationalization for the unknowns, a true useful extract from the sensors, purely imaginary, or somewhere in between. How are we to decide? There is no gauge to compare to absolute knowledge.

We have to assume that the sensors are telling the truth. We have to assume that our model is constructed correctly. We have to assume that the digital computation machine is shifting the zeros and ones properly and not loading them into a wrong register when a cosmic ray zapped it. There is the temptation to discard this simulation as too far a departure into the mathematical shadow world. But there is one form of simulation that has been beneficial for humanity. That is weather forecasting.

Fifty years ago in the 1960's, pilots would not trust a weather forecast more than 24 hours into the future. Today in the 21st century, weather forecasting is fairly accurate a week in advance. How was this possible? It was done with a model of the atmosphere, oceans, and many sensors. The weather is chaotic. In fact, nature is chaotic in general. Weather forecasting is a good model of the mathematics of chaos. Many measurements are taken quickly and transmitted in real time to the model. Adjustments to the model are made in real time. The software learns on the go. The result is most useful for agriculture and for travel planning. Is this real? I would opine that it is better than real because it looks into the future.

Nature

Do other animals count? I know of no other animals that practice mathematics in the form that we use symbols. We do observe animals that move and plan their motions from their sensors of obstacles in their path. A dog's front and hind paws will step over a rock that it saw seconds before. The dog, and other animals, calculate future motion of their muscles. The calculator is analog, resident in their brains. Nature does provide for some capability to do mathematics of locomotion, including for humans. This suggests that mathematics is somehow, on a primitive level, resident within biology and innate. Whether animals count is still unknown, but the higher level of the calculus of motion seems to be there.

The natural inorganic world is chaotic, repetitive, has fractals, and operates on energy. It does not have the geometric shapes of straight lines, perfect circles, and triangles. Humans created them in their structures. In addition, we created them with plenty of symmetry. There is some symmetry in parts of the natural world; in biology, in crystals, and in the microscopic realm. Symmetry is a mathematical construct that both nature and humans have clung to, possibly for reasons to minimize energy.

As mentioned earlier, periodic oscillatory motion is natural. The frequency of that natural motion is a characteristic of it's mass and stiffness. That is the definition of resonance. The amplitude of that natural motion is an equilibrium balance between kinetic and potential energies, which is a consequence of the principle of "least action". Resonance is a highly non-linear behavior. There is no good mathematics to fully quantify that amplified behavior. We deal with resonance from a hodgepodge of measurements, graphs, some trigonometry of waves, differential equations, damping ratios, and simulations. It is worthwhile mentioning that humans have created more resonance than was previously resident in nature by virtue of their desire for symmetry. Flat plates drum, uniform cross sections twang, and homogeneous materials have low damping. It would be better to design machines and structures that break up the symmetry to avoid resonant amplifiers.

Knowledge

Math is heralded as the language of science. Those are not my words, but rather a general understanding that science without math is somehow substandard science. Mathematics provides some comfort in understanding, but it is not true knowledge. Absolute knowledge is based on deductive reasoning. For something to be true, it must be certain, universal, and necessary. In addition, it must be true for all time. These ideas were developed through the logic of mathematics. Modern science, while using mathematics as a language to describe what is observed, takes a different route. Scientists observe something, guess at the operating principles, assign some values to the parameters and arrange them into a convenient formula. Then they make a general conclusion. This is inductive reasoning, arguing from the specific to make a general conclusion; sometimes supported with probabilistic statistics. To say that our knowledge is based on mathematics is false reasoning. The mathematics should be based on knowledge, not the other way around.

Implications for Humanity

Does it really matter if mathematics is an invention or a discovery? Probably not. As an abstract methodology, it is useful to make sense of the surrounding world, whether it is real or not. The numbers enable us to find order among a chaotic world. The numbers are a measure of value. They are also a measure of wealth, however vain that may be.

The ultimate reality is nature. Math is an imaginary caricature of nature to variable degrees of conformance. It is becoming more abstract as time flows forward. Projecting that trend into the future suggest that at some point the creatures that engage deeper into mathematics run the risk of being disconnected from reality.