Action Drama

It is easy to visualize “action” from the above image of a dog going after a toy, but it is difficult to produce a two-dimensional image on graph paper to mathematically describe the concept. Therefore, there will be no graphical image in this article, only words to communicate this abstract idea.

The word "action" invokes" images on many levels. It could be a court proceeding to enforce one's rights to enable justice to be served, sometimes called a motion. It could be an engagement between troops or ships in battle. It could be an operating mechanism of a machine. It could be an opportunity for financial gain, as in a wager or bet. It could be a method of performing, as in the style of movement of the feet. It could be a thing done, like a deed. It could be the unfolding of events in a work of fiction, as suggested in the title above. It could also be something physical, which will be the subject of this article.

Webster's dictionary has definitions of action that are relevant to science[1] --

" The bringing about of an alteration by force or through a natural event."

" The accomplishment of a thing, usually over a period of time."

The first definition above specifies that a force is involved, and the second definition bring in the time element. Both of these physical parameters, force and time, capture the essence of the idea of action. The idea of action was in philosophical discussions prior to the understanding of energy and the laws of motion. It arose from our observations of how nature moves. Many of the initial conclusions were erroneous. For instance, Aristotle taught that a force was necessary to sustain motion. If the force was removed, then the motion of a wagon would cease. That was a valid observation in a real world were friction causes things to come to a stop. In the absence of friction, as in the vacuum of space, force is only necessary to initiate motion or change it's trajectory. Left alone, the object will continue on forever and never stop (theoretically).

Action was the term that described change for centuries prior to 1700. Understanding change was central to philosophical discussions since ancient times. There was no formal scientific definition for quantifying change. It was debated by philosophers on what created change – whether it was natural, mystical, or of divine origin. The motion of celestial objects was attributed to divine intervention. The motion of planets was believed to have nothing to do with gravitational forces, which were unknown in medieval times. Still, action was somehow connected to motion and the continued attention to motion and the attempts to define action eventually led to the better understanding of energy, forces, time, friction, and astronomy. In the 18th century, energy became to be understood as a kind of packet of action. Some effort was needed to initiate change of any kind. Contact force was obvious, but gravity, electrostatics, and magnetism were not clear. Energy was a constituent of action and change, a kind of “packaged dynamism”.

In Newton's Principia, action is mentioned twice in --

Defintion 4: "An impressed force is an **action** exerted upon a body ...."

3rd law: "To every **action** there is always opposed an equal reaction ...."

We all know the word action and the image it produces in the context of the conversation, but can anyone give me a mathematical formula for it?

<u>Units</u>

Action now has a formal definition in physics. It describes how a physical system has changed over time. It has a numerical value that is somewhat abstract with the dimensions of energy x time (joule - sec). This is basically work effort accumulated. For example, if I worked at a production machine for 8 hours, my effort could be given a number as so many pieces output, or as dollars on my paycheck, or as so many watt-hours of electricity consumed by the machine. It would be hard to measure the energy output from me. The time would be easier to measure. If my energy output could be quantified as watts or ft-lbs of effort, then action can be easily calculated as the energy x time.

It is curious, perhaps coincidental, that action has the same dimensional units as momentum and Plank's constant. Action, in physics, being energy x time, has the units of Joule-sec, Ft-Lb_f-sec in the Imperial system, and kg-m^2/sec in the SI system[1]. Angular momentum has the same units, as does Plank's constant. Action is also defined as twice the kinetic energy x time.

Action in physics = energy x time   Joule-sec   kg-m^2/sec   Ft-Lb_f-sec

= 2 x KE x time mv^2-sec   kg-m^2/sec   Ft-Lb_f-sec

= momentum x distance   mv-dist   Ft-Lb_f-sec

Angular momentum = kg-m^2/sec

1 A Joule is equal to 1N-m which is equal to 0.74 Ft-Lb_f .

Plank's constant (k=E/f) = kg-m^2/sec

Plank's constant, 6.626 x 10^{-34} joule-second, may very well be the quantum unit of action.

What, if anything, all this means is discussed later in the philosophy section. For now, action has a scalar value, that is a pure number. It is an overall mathematical description of motion, or alternately, effort.

Principles

One principle concept of action is that in any physical system, the path that an object takes minimizes the action number. For example, if I wanted to go from Albuquerque to Denver, then I could take several routes. The highway route along interstate I-25 will be fast in time. It will consume some amount of fuel and will take about 6 hours for a young driver, somewhat longer for an older and more cautious driver. The two drivers will consume the same amount of energy, but produce different action numbers because of the different times. I could also take the mountain route along Highway 285. That will take longer and consume more fuel going up and down mountain passes. That will produce a different action number again. The scenic view benefit has nothing to do with the action number. It's all about energy and time. Each different route results in a unique number for action, unless the product of energy and time happens to be the same.

This principle of least action was first described by Hamilton and Maupertuis. It states that "nature acts as simple as possible". It suggests minimum entropy generation in accordance with the second law of thermodynamics. However, the principle needs to be qualified. It only works for conservative systems. That is, it will not work when friction is present. Furthermore, it may not be valid for all force laws. In summary, physical processes of motion favor minimizing the action number. It can be minimized as in a rock that does not seem to display any observable change, but the rock is some form of minimum energy state. There may not be any intelligence driving this principle. It's just how the universe works on autopilot with no intelligent interference.

Small sizes favor random motion, like kinetic motion of molecules and random activity of ants. Larger sizes favor directed motion, like heavenly bodies in orbit and elephants walking.

The Math

The mathematical derivation of action is somewhat abstract. It is an excursion into theoretical mathematics. It is beyond the understanding of the average human on the street. We will leave it there.

Energy

Energy is immaterial. It is not anything that you can put in a bottle. It is an abstract generic quantity associated with some kind of change. It is the currency of the physical world. It is the common denominator between mechanical work, gravity, electricity, chemical, and radiation processes. It can be quantified, but only in the context of something moving or changing. It is the essence of process thinking as described by the Greek philosopher Heraclitus.

Let's ponder this action idea from the perspective of energy over some time period. Stars shine. Since our sun continues to pour out huge quantities of radiant energy, it would be a logical conclusion that action will continue to increase unless there is some reverse process that consumes an equal amount of energy to achieve some sort of equilibrium. Otherwise, action will accumulate without bound. Perhaps that energy is converted to mass someplace in the universe and we have a negative action. There was no mention of negative action in the physics definition. It was a scalar number with only a positive sign before the number. Energy radiates outward. Matter diffuses outward. Life expands outward. Let's leave that idea alone for the philosophers to ponder.

If we restrict action to only physical motion, then we can delete the previous paragraph about radiant energy. Positive motion is understandable as a positive number from some starting point, and time is always positive, so action will always calculate to be a positive number. But if we wish to consider both positive and negative motion, then proceed on to the next section.

Oscillation

Motion is relative to something, never absolute. We pick a stable starting point whenever we describe any kind of motion, whether it is unidirectional or oscillatory. This applies to displacement, velocity, acceleration, excursion, or jerk.

When a physical system vibrates, then the motion swings positive and negative. The average motion is zero and there is no positive displacement anywhere. There is a time elapsed, so time is positive, but how to calculate action when the motion turns negative. There is energy even with negative and positive motion swings. Fortunately, there is an energy parameter related to oscillatory motion[2]. The parameter is :

Total Vibratory Energy = $ma^2/2f^2$ where m = mass, kg
a = acceleration, m/s^2
f = frequency, Hz (s^{-1})

The resulting units are $kg\text{-}m^2/s^2$ (mass x velocity squared) which is N-m or Joules.

So action for a vibratory system will have a scalar number associated with the motion. What we do with this is up for grabs. We could tie it to the wear that the machine or structure accumulates. We could use it to describe the unbalance in the system, the misalignment, the state of resonance amplification, or just signal the time for service after so many hours of operation. So action, when calculated for vibratory systems, could have a practical use for machine reliability. Interestingly, action also has the units of joules/frequency.

2 "Vibration Energy", by Victor Wowk, P.E., Vibrations magazine, Winter 2020

Philosophy of Action

The word "action" came ‘front and center’ when Newton introduced the universal law of gravitation. It was described as "action at a distance" by some, and resided in an uncomfortable position among the accepted understanding at the time that contact force was the only force that causes motion. This invisible gravity force was "spooky", as later articulated by Einstein. The same concerns came up later with electric charge and magnetic forces. We still do not fully understand the source of the energy that transmits the force to make something move when exposed to the field, but at least we have a number to quantify the action.

The time parameter of action implies some history. That is just as useful as the odometer in vehicles that accumulates the total trip distance. The duration of time allows for the total effort. Whereas power quantifies the flow of energy, the action quantifies the total expenditure of energy, or the total cost. In fact, we pay for electricity usage in units of watt-hours, or kW-Hrs.

The action gives a panoramic view of the motion. It is an overall picture of the behavior of an object showing possible routes between initial and final situations. In term of the second law of thermodynamics, the action principle has deep implications in relation to entropy. Action is not rational because it cannot be deduced with logic from basic premises.

Action is one of those vaguely understood concepts in physics. It has historical roots with deep philosophical undercurrents. It led to considerations of energy, astronomy, and quantum mechanics. Action now has a specific physics definition with dimensional units that span several diverse fields. For most of us struggling on the planet for survival, it doesn't matter, except for the fact that our perception of the world around us is sensitive to motion.